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MICROFRACTURES, AFTERSHOCKS, AND SEISMICITY

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ABSTRACT

Laboratory investigation of microfracturing in brittle rock has revealed that microfracturing events can be detected after brittle fracture of rock in compression, provided the specimen remains intact. If the sample is isolated after fracture, microfracturing activity decays hyperbolically in a manner similar to typical earthquake aftershock sequences. If reloaded, the sequence is disturbed and a cumulative aftershock pattern develops which is similar to that described by Benioff as strain release due to shear creep. These two types of sequences are discussed with respect to a Markovian model of time dependent fracture in an inhomogeneous brittle medium. This model is then expanded to apply to earthquake aftershock sequences. According to this theory aftershocks are produced by creep rupture due to stress corrosion in the regions of stress concentration following the main shock. The conclusions from laboratory investigations of microfracturing are summarized with respect to the implications regarding the sequence of earthquakes.

Introduction

Detailed laboratory investigations of the deformation and fracture of brittle rock (Mogi, 1962a, 1962b, 1963a, 1963b; Scholz, 1968a, 1968b, 1968c, 1968d) have shown that such deformation is accompanied by small scale cracking (microfracturing) which produces radiative elastic energy in a manner analogous to earthquakes. This microfracturing process is responsible for most of the inelastic deformation of rock and appears to be an inherent property of inhomogeneous brittle materials. The interesting thing from the seismological viewpoint is that earthquake behavior is quite similar to microfracturing statistically, which implies that seismicity may be a result of this sort of process occurring on a much larger scale within the crust.

One of the most distinctive of earthquake phenomena is the aftershock sequence. Almost all crustal earthquakes, regardless of size, appear to be followed by a series of smaller shocks, or aftershocks. Aftershocks are usually strongly related to the source region of the main shock, but the spacial patterns within the aftershock region are quite varied for different sequences. In contrast, the time sequence of aftershocks is consistently similar for all sequences, suggesting that aftershock sequences may be produced by a well defined gross process.

The universal observance of aftershock sequences in the Earth suggests that, if microfracturing is truly similar to seismicity, some sort of aftershock sequence should be reproducible in the laboratory. The experimental difficulty, of course, is that laboratory fracture specimens are of finite size and do not remain intact after failure. This effect is thought to be due to the testing machine, which imparts a large amount of energy to the specimen during failure resulting in its total destruction. It is possible however to maintain the sample in an intact state after fracture if the testing machine employed is very stiff relative to the sample. In the present investigation this was done and it was found that an aftershock sequence could be observed following fracture of rock in compression.

These observations allow speculation on the nature of aftershock sequences with a degree of confidence, since knowledge of the properties of rock in the laboratory and the conditions under which the experiments were performed produces some insights into the aftershock process. The similarity of laboratory produced aftershock sequences to those occurring naturally reenforces some of our earlier conclusions regarding the similarity of crustal deformation and microfracturing. Further discussion of the nature of crustal deformation and seismicity is made in this context.

Observations

The experiments were performed by stressing specimens to fracture in uniaxial compression and monitoring microfracturing activity for a period of time following fracture. The testing apparatus was a ball screw type machine with a stiffness of 10^6 kg/cm. Specimens were 1.5 cm diameter dogbone shaped samples of the design by Mogi (1966). The rocks tested, Westerly granite and San Marcos gabbro, have been described previously (Scholz, 1968a).

The technique of monitoring microfracturing events is the same as that used in earlier studies (Scholz, 1968a). Briefly, the events are detected with a piezoelectric transducer, amplified, shaped, and counted with a multichannel pulse height analyzer used as a multiscaler. A 100 channel analyzer was used in some of the experiments. The dwell time used in those experiments was 1 sec, so that a record was obtained of the number of events that occurred in successive 1 sec intervals for 100 sec after fracture. In other experiments a 400 channel analyzer was employed with a 5 sec dwell time in order to obtain a broader time window.

The testing machine consists of a ball screw which is loaded on top and is slowly lowered (2×10^{-5} in/sec) on the specimen by means of a constant speed motor drive. In a number of the experiments the motor drive was shut off immediately after fracture of the specimen. At that stage a small air gap existed between the specimen and the piston, so that there was no load on the sample during the observed aftershock sequence. Although the samples had nearly throughgoing faults of the typical acutely inclined compressional type, they were still intact in that they had finite tensile strength and were capable of being handled without breaking into several parts. The fault usually showed a small displacement of a few tenths of a millimeter. Numerous extension cracks up to a few millimeters in length usually extended in nearly axial directions from the fault surface.

Typical results for Westerly granite and San Marcos gabbro are shown respectively in Figures 1a and 1b. These are semilog plots of the accumulated number of microfracture events detected versus time after fracture. Notice that the curves are quite linear, indicating that frequency decays as the inverse of time as it does often in earthquake aftershock sequences. The curves also show some fine structure, which is also typical of aftershock sequences in the Earth. The possible significance of this fine structure will be discussed below.

The experiment was repeated several times with Westerly granite. In each case the cumulative frequency N of microfractures during the aftershock sequence was well approximated by the relation

$$N = a + b \log \tau \tag{1}$$

as outlined above, where a and b are constants and τ is the time after fracture. It is interesting, however, that a and b are notably not reproducible between experiments. This indicates that these constants are not properties of the material, but depend on the detailed configuration of the fault after failure, as might be expected.

During several of the experiments the motor drive was not turned off after fracture. The aftershock behavior in this case was of a dual nature as illustrated in Figures 2a and 2b. Initially, the aftershock rate decays according to (1) which we

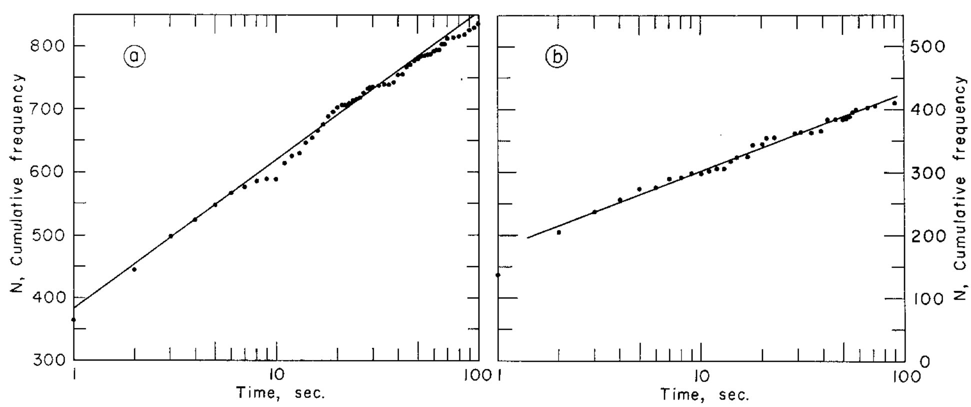

FIG. 1. The cumulative frequency of microfracturing events detected versus time after fracture, a) Westerly granite, b) San Marcos gabbro. The specimens were stress-free during these measurements.

shall refer to as Type I, but after a short period of time a very sharp increase of activity occurs. In the latter stage (Type II) the accumulated frequency of aftershocks can be described by the relation

$$N = \alpha + \beta[1 - \exp(-\gamma\, t^{1/2})] \tag{2}$$

where t is the time measured from the beginning of the second stage. The second stage begins when the ram comes once again into contact with the specimen. In these latter experiments in which the ram was allowed to continue its descent, for a short period of time the specimen was free of the machine and the normal aftershock behavior occurred. The ram then contacted the specimen and proceeded to drive it at a constant rate, producing the second stage type of sequence.

Discussion and Theory

The experimental results have shown that elastic signals are radiated from rock after brittle fracture in compression, provided that the body remains intact. The signals which were observed are very similar to those attributed earlier (Scholz; 1968a, 1968c) to microfracturing, i.e., the propagation of small cracks within the body. Apparently when fracture occurs in rock a situation is produced such that fracturing occurs on a scale small relative to that of the main fracture. The most striking feature of this behavior is that the postfracture microfracturing activity decreases at a characteristic hyperbolic rate provided that the system is isolated mechanically. The detailed configuration of the fault, which is likely to vary between experiments, seems to affect only the decay constants and not the form of the decay law. This itself suggests that the principal features of the sequence must be regulated by a fairly general process.

For the present we shall refer to the sequence of microfracturing which occurs after failure as an aftershock sequence though making it clear that we have not

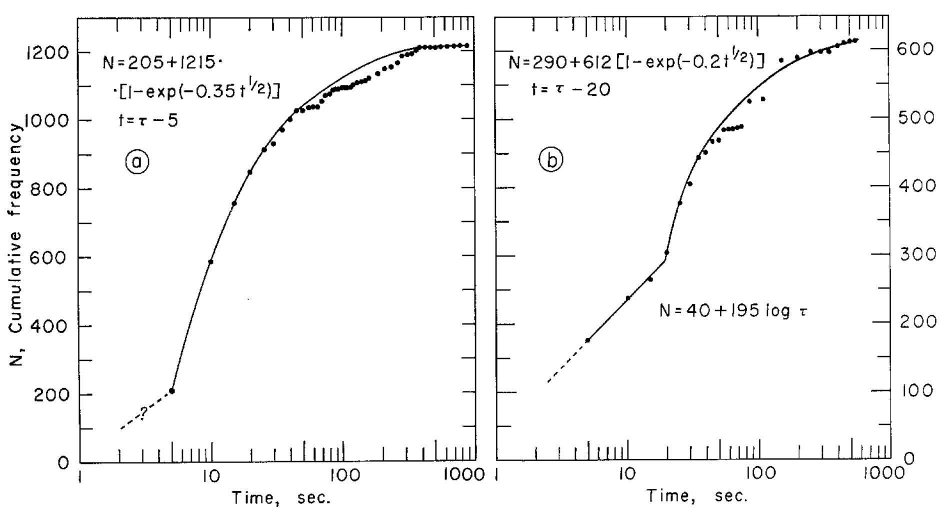

FIG. 2. The cumulative frequency of microfracture events vs. time after fracture for specimens that were reloaded. a) San Marcos gabbro, b) Westerly granite.

yet established the relationship between this behavior and that which follows an earthquake. We shall presently be discussing only the Type I aftershocks which were observed in the laboratory. The Type II sequence is probably a hybrid process produced by the influence of reloading on the Type I sequence.

Time dependent microfracture sequences such as we are discussing are a rather fundamental property of brittle rock. When an intact rock is stressed to some level which is held constant, microfracturing activity occurs which decays hyperbolically with time. Such microfracturing produces small increments of strain which have been found to be responsible for the creep of brittle rock at low temperature and pressure. This process of creep produced by a time dependent microfracturing mechanism can be explained in a rather simple way by considering the properties of an inhomogeneous brittle material composed of homogeneous elements with time dependent strengths (Scholz, 1968b).

Such an inhomogeneous medium can be described by considering the probability

$f(\sigma; \bar{\sigma})$ that the stress at a point has some local level σ when the mean (or applied) stress is $\bar{\sigma}$. We are thus treating the local stress as a random variable with mean $\bar{\sigma}$. Most brittle materials, particularly the silicates, have a strength which is time dependent, i.e., if the material is stressed to some value which is held constant, fracture will occur at some time after application of the load. The time required for fracture varies in an inverse way with the stress. Time dependent fracture of this nature is termed static fatigue or creep rupture. If we consider that the strength of regions (small enough such that the local stress σ on them can be considered uniform) has this property, then in an inhomogeneous medium time dependent microfracturing will occur after application of a stress. If the microfracturing events can be considered to be randomly independent, the activity at any time t after application of stress is dependent only on the current stress distribution $f(\sigma; \bar{\sigma}, t)$, and not on the prior history of microfracturing. Therefore we can describe the microfracturing sequence as a Markov process with a stationary transition probability of fracture $\mu(\sigma)\, dt$. The microfracturing frequency n can then be computed from the basic Chapman-Kolmogorov equation for the process

$$n = \int_{S(0)}^{\infty} f(\sigma; \bar{\sigma}, t)\mu(\sigma)\, d\sigma \tag{3}$$

where $S(0)$ is the initial mean local strength. In order to calculate the stress distribution at time t, $f(\sigma; \bar{\sigma}, t)$, from the initial distribution $f(\sigma; \bar{\sigma})$, a property of the material, we apply the constraint that each region fails only once. That is

$$\frac{\partial}{\partial t} f(\sigma; \bar{\sigma}, t) = -f(\sigma; \bar{\sigma}, t)\mu(\sigma). \tag{4}$$

Equations (3) and (4) allow the calculation of the time sequence of fracturing within the material provided that we know the time dependence of strength of the individual homogeneous elements and that we can approximate the initial stress distribution. For rock on the laboratory scale, the individual elements are single crystals. The time dependence of the strength of silicate single crystals in the brittle range can be approximated by the empirical formula (Scholz, 1968b)

$$\langle t \rangle = k \exp\left[\frac{S(0) - \sigma}{c}\right] \tag{5}$$

where $\langle t \rangle$ is the mean time to fracture of an element at stress σ and k and c are constants which depend on the material, the temperature, and the environment.

For a Markov process the transition probability is related to the mean recurrence time by

$$\mu(\sigma)\, dt = dt/\langle t \rangle$$

so that

$$\mu(\sigma)\, dt = \frac{1}{k} \exp\left[-\frac{S(0) - \sigma}{c}\right] dt. \tag{6}$$

Integrating (4) we have

$$f(\sigma; \bar{\sigma}, t) = f(\sigma; \bar{\sigma}, 0) \exp [-\mu(\sigma)t] \tag{7}$$

and from (6) the relation

$$d\mu(\sigma) = \frac{1}{c} \mu(\sigma) \, d\sigma. \tag{8}$$

Substituting (6), (7), and (8) into equation (3) we can solve for the time development of the process. Assuming that the initial distribution $f(\sigma; \bar{\sigma}, 0)$ is a constant M over the range $S(0) \leqq \sigma \leqq 0$ and zero for $\sigma \geqq 0$, (compressive stress taken as negative), we obtain

$$\begin{aligned} n &= Mc \int_{S(0)}^{0} e^{-\mu(\sigma)t} \, d\mu(\sigma) \\ &= \frac{Mc}{t} \int [e^{-t/\langle t \rangle_0} - e^{-t/\langle t \rangle_{S(0)}}] \end{aligned} \tag{9}$$

where $\langle t \rangle_{S(0)}^{-1} = \mu(S(0)) = 1/k$ and $\langle t \rangle_0^{-1} = \mu(0) = 1/k \exp [-S(0)/c]$. In the range of interest, $\langle t \rangle_{S(0)} \ll t \ll \langle t \rangle_0$, the factor in brackets is essentially unity, and

$$n = \frac{Mc}{t} \tag{10}$$

or

$$N = Mc \log t + b. \tag{11}$$

Equations (10) and (11) give the time sequence of microfracturing during transient creep of rock. This result is typical of exhaustion theories of creep. The decay law depends primarily on the stress dependence of the mechanism (equation 5) and therefore does not uniquely determine the mechanism. Any thermally activated mechanism which is linearly dependent on stress will produce a hyperbolic decay. Under shallow crustal conditions, however, time dependent microfracturing due to static fatigue has been demonstrated experimentally to be responsible for creep in silicate rock (Scholz, 1968b).

It seems clear that the aftershock sequences which we have observed must be somehow related to transient creep since both processes involve time dependent microfracturing characterized by precisely the same sort of statistical decay law. Although the mean stress drops to a very low value after fracture (indeed, to zero in the experiments), high stresses must exist somewhere within the body in order for such microfracturing to occur. The situation can be visualized as shown in Figure 3. The diagram shows the statistical stress distribution schematically both before and after fracture. Just prior to fracture the mean stress is at $\bar{\sigma}$ and the stress distribution is given by the density function $f(\sigma; \bar{\sigma}, t)$ shown in the figure. Here we show a more realistic stress distribution than the constant distribution we assumed in the analysis. However unless $f(\sigma;\,\bar{\sigma},\,0)$ varies as steeply with σ as $\exp\,[-\mu(\sigma)t]$, it cannot significantly affect the creep law (Cottrell, 1953, p. 201). This condition is considered to be always true since Cottrell has shown (Davis and Thompson, 1950) that $\exp\,[\mu(\sigma)t]$ is practically a step function centered at a value $\tilde{\sigma}$ such that $\mu(\tilde{\sigma})\ t = 1$. At time t almost all regions in which the stress is greater than $\tilde{\sigma}$ will have failed whereas those at lower stress will have not failed. Therefore $\tilde{\sigma}$ can be defined as the mean local strength at time t, $S(t)$. In Figure 3, the stress distribution is therefore truncated to the left of $S(t)$ as it decreases (sweeps to the right in the figure) during creep.

When fracture of the entire body occurs, the mean stress drops to some new value $\bar{\sigma}'$ (in the experiments, $\bar{\sigma}' = 0$) but regions of very high stress are produced

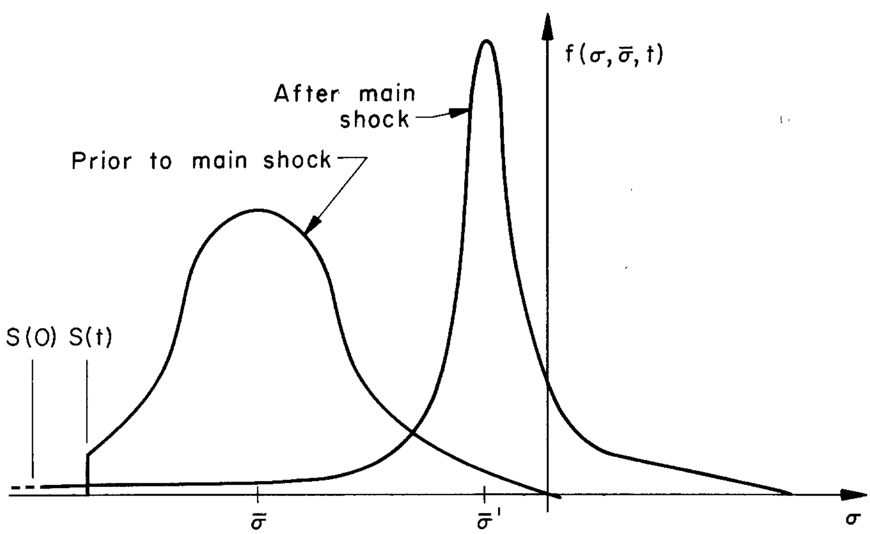

FIG. 3. A schematic representation of the statistical stress distribution before and after fracture. Compressive stresses are taken as negative.

which are represented by the probability mass in the tails of a new stress distribution function $f'(\sigma;\,\bar{\sigma}',\,t)$ shown in the diagram. The new regions of stress concentration will not in general be the same as those prior to fracture, so they may be characterized by the initial (unweakened) strength $S(0)$ which decays in time, producing microfracturing. According to this view, then, aftershocks can be considered to be produced by a process identical to that responsible for creep. The process is now occurring within a material with a new stress distribution, $f'(\sigma;\,\bar{\sigma},\,t)$, so by substituting this distribution into equations (3) and (4), the time sequence of aftershocks given by (10) and (11) is obtained.

There is a significant difference, however, in the type of stress distribution in the fractured and unfractured material. Except for a short interval just prior to fracture, regions of high stress in unfractured rock, and hence sources of microfractures, are uniformly distributed throughout the body (Scholz, 1968c). On the other hand, high stress regions in the fractured rock must somehow be closely related to the fault. In the present experiments, although the fault formed a roughly defined plane through the specimen, it is on close inspection a fault zone having a maximum width of a few millimeters and defined by a number of semiparallel auxiliary faults. Extending from the fault are numerous small cracks and extension fractures. It is likely that regions of high stress and hence aftershocks are concentrated within this narrow disordered band near the fault. The stress distribution $f'(\sigma; \bar{\sigma}, t)$ which controls the process therefore is only defined within this band and hence the aftershock process is in a sense a two dimensional analogue of creep. There is some indirect evidence to verify this viewpoint. Microfracturing has been studied during stick-slip in specimens containing pre-cut sliding surfaces (Scholz, 1968a). In that case, sliding occurs without the production of a substantial disturbed zone, and no aftershocks are detectable.

Thus far we have described the aftershock sequences which were observed in the laboratory and have attempted to analyze the determining mechanism from a consideration of processes which are known to occur in rock. This analysis simply consisted in suggesting that the state of stress in a body following fracture is similar to that produced in a creep test in that regions of high stress are present and boundaries are held at constant stress. The real problem which now comes to hand is to relate this with the similar process of earthquake aftershocks in order to attempt to clarify the latter.

Aftershock sequences have long been a major area of research in seismology, and a number of general properties can be attributed to them. Shallow tectonic earthquakes, regardless of their size, are almost always accompanied by aftershocks, whereas the same is not true of deep earthquakes (Isacks *et al*, 1967). The spacial extent of aftershock activity is normally closely related to the region faulted during the main shock. Generally the activity appears to be distributed through the faulted area although sometimes it is concentrated in regions such as the ends of the fault (e.g., Benioff, 1956a). Such concentration of aftershock activity as well as phenomenological arguments such as were presented above with reference to the laboratory experiments have led a number of workers (e.g., Mogi, 1962a) to suggest that aftershocks must result from stress concentrations produced by the main shock.

The time statistics of aftershocks are well known and can usually be well approximated by the hyperbolic relationship which we have observed in the laboratory. Utsu (1961) has suggested in more general law of the form

$$n = \frac{b}{(d + t)^h} \tag{12}$$

where h usually varies from about 0.9 to 1.3. A review of his data plus that of Mogi (1962a), however, demonstrates that in most cases the exponent h cannot be differentiated from 1 due to scatter in the data. Although there may be important exceptions, earthquake aftershock activity generally decays nearly hyperbolically.

In a related study (Scholz, 1968d) the frequency-magnitude relation of microfracture events was studied and found to be very similar to that of earthquakes. Basic considerations of the fracture propagation properties of a brittle inhomogeneous body demonstrated that this sort of frequency-magnitude relation is a fundamental property of such a material. Therefore the statistical similarity of microfracturing and earthquakes is not simply fortuitous but is a result of the underlying similarity of rock in the laboratory and in the scale of the Earth's crust. The significant difference is simply in the scale of the inhomogeneities of the stress field. From the present analysis, the occurrence of aftershocks can be understood from the viewpoint of the laboratory results as well. Given the realistic assumptions that stress concentrations result from faulting, aftershocks will occur in a general inhomogeneous brittle material with time dependent strength in a sequence described by equations (3) and (4). At that stage of the analysis only general statistical properties such as the independence of the events was introduced so the results can be considered equally applicable to either rock in the laboratory or on the scale of the crust. The derived time sequence, however, depends upon the physical mechanism resulting in a time dependent strength, as expressed by equation (5). The form of (5) used in the present analysis is an approximation of the empirical static fatigue behavior of a wide variety of homogeneous silica based systems such as glasses. The mechanism of the static fatigue process in such systems is thought to be weakening by stress corrosion. The rates of at least some chemical reactions involving solids are dependent upon the mechanical stress levels at or near the reacting surfaces. Such reactions can modify the local stress levels and introduce time dependencies in the strength. The primary reactions which weaken silicates are hydration reactions. The reaction in brittle materials under stress is accelerated by high tensile stresses at the tips of cracks and serves to lengthen the cracks, thus weakening the material (Charles, 1959; le Roux, 1965; Hillig and Charles, 1965). Even in the plastic range of temperature and pressure these reactions are known to weaken silicates substantially by mobilizing dislocations (Griggs, 1966).

Although the experiments on microfracturing duplicate to a certain extent the condition that exists in regions of shallow earthquakes, the question of scale leads to difficulties in applying the mechanism found to produce earthquake-like sequences in the laboratory to that of real earthquakes. It was pointed out with regard to the frequency-magnitude relation that the basic statistical similarities are due to the fundamental properties of inhomogeneous brittle systems and are insensitive to the detailed mechanisms involved. However, in the case of aftershocks, the time decay law depends on the mechanism and is the same for earthquakes and microfractures. This indicates the time dependence of strength of substructural units in the crust is of the form (5), implying that the strength of rock as a whole has the same type of time dependence as the component grains. This suggestion requires some justification. We have established that time dependent microfracturing occurs in rock in the laboratory and is likely due to static fatigue of individual grains by stress corrosion. It also seems reasonable to expect that this will be the primary microscopic mode of time dependent deformation of rock *in situ* under similar conditions of temperature and pressure. In discussing earthquakes, however, we must consider whole rock as the individual elements in the model. The fracturing process of brittle rock in constant strain rate experiments appears to be due to the cumulative buildup of microfractures which produces an instability (Scholz, 1968a, 1968c; Brace and Orange, 1968) when the microfractures reach a critical density. Thus rock fracture appears to be due to a geometrical instability in much the way tensile strength in metals is related to necking. Since such instabilities are basically independent of prior history, the criterion of critical microfracture density should also be applicable to creep rupture of brittle rock—the analogous process in the time domain. Noting the criterion indicates that the cumulative frequency N and thus the distribution function $F(S(t); \bar{\sigma}, t)$ are constants at fracture, it is implied that

$$C(t) = \rho S(t) \tag{13}$$

where $C(t)$ is the fracture strength of the rock at time t and ρ is a material constant. Equation (13) implies that the central moments of the stress distribution are no more than linear in $\bar{\sigma}$. Experimental results (Scholz, 1968a) indicate that the mean and standard deviation are linear in $\bar{\sigma}$ in most rocks. We can now calculate the time dependence of strength from $S(t)$, the mean local strength in the case of constant $\bar{\sigma}$. Consider again $S(t)$ as a step function advancing in time along the σ axis as in Figure 3. To determine the rate of advance we set

$$\mu[S(t)]\, t = 1 \tag{14}$$

and substituting from equation (6)

$$\mu[S(t)] = \frac{1}{t} = k \exp\left[-\frac{S(0) - S(t)}{c}\right] \tag{15}$$

or, equivalently, combining (13) and (15)

$$t = \frac{1}{k} \exp\left[\frac{S(0) - \rho C(t)}{c}\right] \tag{16}$$

which is a relation between time and stress at fracture for whole rock which is of the same form as the equivalent relation for the individual grains given in (5). Mogi (1962b) studied creep rupture in granite over several orders of magnitude in time with results that support (16).

Perhaps not unexpectedly, then, creep rupture of rock follows a time law which is similar to that followed by the microscopic mechanism involved. In this light we can substitute equation (16) for (5) and calculate the time characteristics, (10) and (11), for an ideal earthquake aftershock sequence.

Having made some justification for the applicability of the experimental results toward interpretation of earthquake aftershock sequences, it is interesting to note that both the Type I and II sequences have been observed naturally. Benioff (1951) observed these two types of strain release sequences and interpreted them as being due to relaxation under two types of stress systems. He later pointed out, however, (Benioff, 1955b) that the interpretation is equally valid for forward creep. In the experimental sequences, the process cannot be due to relaxation since in the case of Type I the boundaries are stress-free, whereas in the Type II sequence deformation is constrained to be in the forward direction. The idea that aftershocks produce creep in the forward direction is supported by evidence that aftershocks often have the same mechanism as the main shock (Stauder and Bollinger, 1966) and that creep during an aftershock sequence is in the forward direction (Wallace and Roth, 1967; Smith and Wyss, 1968).

The analysis indicates that aftershocks in an isolated (constant stress boundary conditions) body should decay hyperbolically. This is broadly true of many aftershock sequences although significant deviations over short periods of time such as observed experimentally (see Figure 1) may reflect that in detail events are not truly independent locally. Sharp changes from Type I to Type II sequences such as observed by Benioff may be due to reloading of the faulted block as is the case in the laboratory. Alternatively, rapid changes in the environment or in the material properties could produce sharp changes in the time sequence. Such anomalous effects on the afterworking of metals subjected rapidly to a corrosive environment have been noticed by Barrett *et al* (1953).

Conclusions

Mogi's investigations of microfracturing revealed that the process of microfracturing in laboratory specimens is similar in many statistical respects, albeit on a much smaller scale, to the earthquake generating process. In the light of this evidence he suggested that microfracturing is a scale model of seismicity. In the present studies a number of new aspects of microfracturing have been observed and the physics of the process described theoretically. This work has brought out the physical basis for the similarity of microfracturing and seismicity, which does not lie in exact similarity of the mechanisms responsible for the two processes but on more general similarities of the properties of rock in the laboratory and on the scale of the earth's crust. These concepts yield a rather simple understanding of the general pattern of seismicity.

The basis of this similarity is that rock under shallow crustal conditions is a brittle material in its mode of fracture. The brittle fracture of rock is not, however, a discrete but a cumulative process involving a complex history of small scale fracturing which leads to overall fracture. This behavior is due to the elastic inhomogeneity of the material which produces fluctuations in the stress field. Thus, local fracturing takes place progressively during the loading history at points of diverse stress concentration and is stabilized by the presence of troughs in the stress field. The scale of such inhomogeneities may range from intergranular to geologic.

We can describe the stress σ at a point in such an inhomogeneous body as being composed of a deterministic part s and a random or statistical part r, thus

$$\sigma = s + r \tag{17}$$

where r may be of the order of s. The s and r parts may be identified with tectonic loading and structure, respectively. Local fracture occurs at points where σ exceeds the strength $S(t)$, a function of time. Suppose this material is stressed at a uniform rate. At an early stage of the loading history, if the material is statistically uniform, local fractures will occur in isolated regions and are independent events. The random part of the stress r, and therefore σ in a uniform field, can be considered approximately normally distributed, and fracturing at constant stress Markovian. In other words, at this stage the autocorrelation function of stress in space, given by

$$R_{\sigma\sigma}(d) = \lim_{D\to\infty} \frac{1}{2D} \int_{-D}^{D} \sigma(d)\sigma(d + \delta)\, d\delta \tag{18}$$

where d is a position coordinate, can be approximated as a delta function. At this stage the events are distributed uniformly throughout the body.

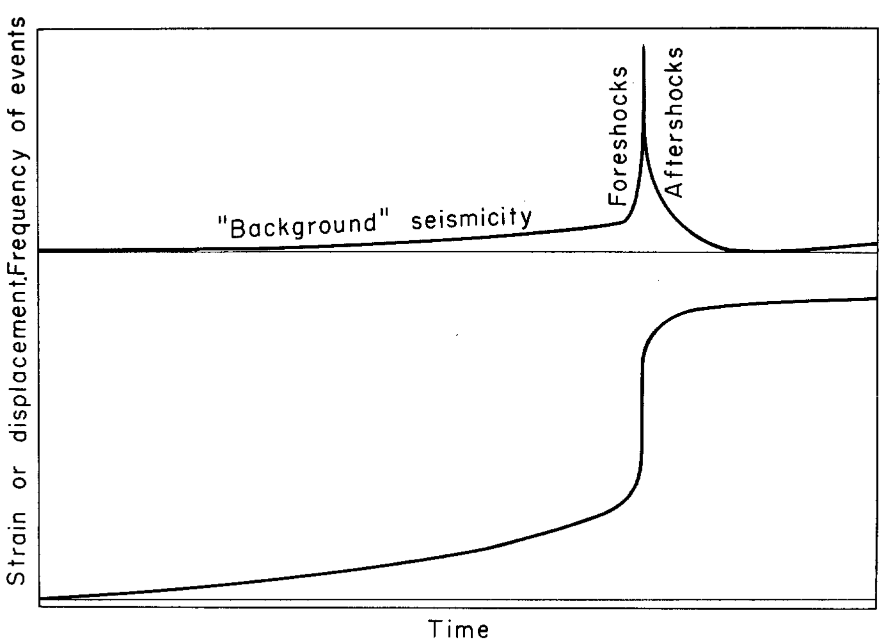

Fig. 4. The response of an inhomogeneous brittle material to stress increased at a constant rate. Upper figure: the time sequence of local fracturing events. Lower figure: the displacement (or strain) pattern developed during the sequence.

The approximation of the form of (18) is appropriate if the events sampled are isolated from one another in space. Therefore the present analysis is only applicable to earthquake statistics averaged over a substantial region. We may view it as a special case of a wider family of stochastic processes regulated by the same physical processes but modified by local structure. One way to describe the influence of local structure is to maintain the concept of distance as a time-like parameter and consider the power spectrum $\varphi_{\sigma\sigma}(\omega)$, defined as the Fourier transform of $R_{\sigma\sigma}(d)$. The function $\varphi_{\sigma\sigma}(\omega)$ contains most of the information regarding the influence of local structure in the process. In the special case we have chosen, the power spectrum is white. If the region chosen for study is small enough, however, this will in general not be the case and the statistics, particularly the frequency-magnitude relation, will be profoundly affected. The effect of periodicity in structure on the frequency-magnitude relation has been discussed qualitatively by Mogi (1967).

Eventually the local fractures become close enough together so that they are no longer independent and an instability develops which leads to the large scale failure of the body. Prior to total failure there is a rapid increase of local fracturing activity and the events cluster in a zone in which failure eventually occurs (Scholz, 1968c). These various stages are illustrated schematically with an arbitrary time scale in Figure 4. These stages, including the aftershock sequence described above, are observable in rock deformed in the laboratory. As outlined in the figure, the behavior is identifiable with the various stages of seismic activity leading up to, and following, a major earthquake.

Quite generally, then, microfracturing does appear to be a scale model of seismicity, and the model which has been developed to describe the behavior shown in Figure 4 is equally applicable to both cases. In regions such as California, however, where many such sequences have occurred in the past, a well-defined fault has been developed which concentrates earthquake activity. Such a system must be viewed two dimensionally, rather than three dimensionally such as we have discussed.

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